ADAPTIVE WAVELET-ALGORITHMS AND BUCKLING OF INELASTIC SHELLS

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During inelastic deformations of thin structures the stresses exhibit a complicated and a priori unknown behavior across the thickness of the structure which makes it difficult to establish numerical algorithms which are efficient, accurate and stable. High accuracy, however, is indispensible in the case of structural instabilities, which are extremely sensitive on computational and modelling errors. In this presentation we will deal with these problems using wavelet–expansions of the stresses across the thickness of the shell. Let us assume an expansion of the strain in powers of the coordinate ξ normal to the shell–midsurface which means that we have to compute stress resultants as thermodynamically conjugate quantities, see [1]. This yields

$$\mathbf{E} = \sum_{n=0}^{N} \mathbf{E}^n \xi^n, \qquad \mathbf{S}^n = \int_{-h/2}^{+h/2} \mathbf{S}(\mathbf{E}, \mathbf{K}, \xi)\, \xi^n d\xi, \tag{1}$$

where $\mathbf{K}$ can be an arbitrary internal variable, e.g. plastic strain, a hardening parameter or a damage variable. For nonlinear materials these integrals cannot be evaluated analytically in general and one has to resort to a numerical quadrature scheme:

$$\mathbf{S}^n \approx \sum_{i=1}^{K} \alpha_i\, \mathbf{S}(\mathbf{E}(\xi^i), \mathbf{K}(\xi^i), \xi^i)\, (\xi^i)^n. \tag{2}$$

Here ξ^i denote collocation points and α_i weight factors. The effective evaluation of Eq. (2) constitutes a crucial part of any algorithm for nonlinear shells. On the one side it has to be sufficiently accurate, on the other side it is known that employing too many collocation points can cause numerical instabilities.

In this paper we will suggest an adaptive integration scheme based on wavelet–expansions of the stresses:

$$\mathbf{S} = \sum_j \bar{\mathbf{S}}^{0,j}\, \varphi(\xi - j) + \sum_{i,j} \mathbf{S}^{i,j}\, \psi(2^i \xi - j). \tag{3}$$

The adaptive algorithm relies on the following feature of wavelet expansions. If $\mathbf{S} \in \mathcal{C}^n$, i.e if it is n–times continuously differentiable, then the expansion coefficients decay like $\mathbf{S}^{i,j} \sim 2^{-(n+1)i}$. This property holds locally, i.e. it can be used to detect irregularities in the stress behavior and to estimate the error of the approximation (2). Numerical examples for different inelastic materials will be discussed.

References

[1]K. Hackl, “A Framework for Nonlinear Shells Based on Generalized Stress and Strain Measures”, *Int. J. Solids Structures*, v. 34, p. 1609-1632, 1997.